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An Examination of Fourth and Fifth Graders' Fractional Understandings Based on Mathematical Achievement

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ABSTRACT

The purpose of this experimental study is to investigate whether low, middle and high achieving students could benefit at the same extent from a fraction instruction which was prepared according to basic principles of Socio-constructivism and Realistic Mathematics Education. To this end, an instruction starting with sharing situations, and focusing on group and class discussions was carried out with experimental group. Meanwhile, the students in the control group attended their regular lessons. Both groups were consisted of 27 fourth and 28 fifth grade students. Three tests were administered to the participants: General Mathematical Achievement Test, Fractional Understanding Pre Test and Fractional Understanding Post Test. According to t-test and ANCOVA results, the positive effect of the designed learning environment on fractional understanding of high, middle and low achieving students was substantially similar. Likewise, high, middle and low achievers in the control group also did not show any difference with regard to effect of the traditional learning environment on their fractional understanding.

Key Words: Fractional Understanding, Fraction Learning, Mathematical Achievement.

Dört ve Beşinci Sınıf Öğrencilerinin Kesir Kavrayışlarının Matematiksel Başarılarına Göre İncelenmesi

ÖZET

Bu deneysel çalışmanın amacı, düşük, orta ve yüksek başarılı öğrencilerin, Sosyokonstrüktivizm ve Gerçekçi Matematik Eğitimi'nin temel ilkelerine göre hazırlanan bir kesir eğitimden aynı düzeyde yararlanıp yararlanamayacaklarını incelemektir. Bu amaçla, deney grubuna paylaşım durumlarıyla başlayan, sınıf ve grup tartışmalarına odaklanan bir eğitim verilmiştir. Bu arada kontrol grubundaki öğrenciler normal eğitimlerine devam etmişlerdir. Her iki grup 27 dördüncü ve 28 beşinci sınıf öğrencisinden oluşmuştur. Katılımcılara 3 test uygulanmıştır: Genel Matematiksel Başarı Testi, Kesir Kavrayışı Ön Testi ve Kesir Kavrayışı Son Testi. T-testi ve ANCOVA sonuçlarına göre, tasarlanan öğrenme ortamının düşük, orta ve yüksek başarılı öğrencilerin kesir kavrayışları üzerindeki olumlu etkisi büyük oranda benzerdir. Aynı şekilde, kontrol grubundaki düşük, orta ve yüksek başarılı öğrenciler de geleneksel öğrenme ortamının kesir kavrayışları üzerindeki etkisi açısından bir farklılık göstermemişlerdir.

Anahtar Sözcükler: Kesir Kavrayışı, Kesir Öğrenme, Matematiksel Başarı.

INTRODUCTION

A long history of research and curriculum development efforts reveals that fraction teaching and learning remain a major challenge for teachers and students (Watanabe, 2007). The acknowledged difficulties in learning fractions are reflected and documented in a number of studies in which researchers have examined different aspects of this topic (eg. Hart et al., 1981; Haseman, 1981; Behr et al., 1984; Post et al., 1986). More recent studies by Mack (1998); Tzur (1999) and Anderson, Anderson and Wensell (2000) revealed that understanding and using fractions are tasks that have traditionally been difficult for pupils. National and international assessment results showed that even older pupils had trouble in working with and understanding fractions (NCTM, 1989; Mullis et al., 1997).

Despite learning difficulties about fractions and advantages of using decimals and percentages, they still deserve an important place in primary education. The first reason is that we often figure and think in terms of fractions, even when fractions are not explicitly involved. For example, if we

have to estimate 72% of 600, we probably figure that 72% is about three-quarters. That is, fractions play an important role in mental arithmetic when percentages and decimals are involved. The second reason is didactic in nature: If you understand fractions, you have a good foundation for proportions, decimal numbers and percentages (NCTM, 2000; Galen et al., 2008)

For having a general overview about experimental studies related to teaching of fractions, we can summarize the results of Pitkethly and Hunting (1996)'s analysis in the current study. The authors reviewed research in the area of initial fraction concepts. They found that the common goal of the empirical studies was to assist children to develop a meaningful understanding of the rational number construct that was founded on durable fraction concepts. According to the authors, two interpretations of findings were derived from the research. One group of researchers identified initial fraction concepts emerging from the application of intuitive mechanisms, in particular partitioning in either continuous or discrete contexts, and leading to unit identification and iteration of the unit. The other group of researchers identified ideas of ratio and proportion that existed in young children's early thoughts about fractions.

Within empirical studies, the two of them by Streefland (1991) and Keijzer (2003) have special importance since their contribution to theoretical framework and methodology of the present study. Therefore, there is a need to give more detailed information about these studies.

Leen Streefland (1991) implemented a teaching experiment which lasted two year with sixteen primary-school children. The goals of the project were to develop a fraction program, and to produce a theory on teaching and learning of fractions which would be an example of theory of Realistic Mathematics Education. He provided numerous instructional examples of intuitive, fractional knowledge building that let participants to construct their own understanding of fractions and procedures for the operations. The activities recommended by Streefland linked fractions and ratios during the learning process. In order to attribute solution procedures and results in the experimental group to the realistic program, the learning processes of these children were compared to the learning processes of other groups of students who were instructed in alternative fraction programs. These programs largely focused on rules and algorithms. There were considerable differences in the achievements of the two groups. The control group had no advantage in calculation. The experimental group scored higher than the control group whenever the problems required a solution without observable, mathematical or visual aids.

Keijzer (2003) conducted a longitudinal study of teaching and learning the subject of fractions in two matched groups of ten 9–10-year-old students. Both groups were subjected to instruction based on Realistic Mathematics Education. In the experimental group fractions were introduced using the bar and the number line as (mental) models, while in the control group the subject was introduced by fair sharing and the circle-model. In the experimental group students were invited to discuss, in the control group students worked individually. The groups were compared on several occasions during one year. After one year, the experimental students showed more proficiency in fractions than those in the control group.

The study of Woodward and Baxter (1997) is worth to discuss in the present study in the sense of analyzing an experimental instruction's effects on understanding of pupils at different achievement levels. They examined students' learning when using an innovative mathematics curriculum, and they found that when students were separated into three achievement categories, students in the middle group exhibited the highest improvement while students in the lowest group had a level of improvement that was not statistically significant. In addition, when we thoroughly revise the literature in terms of investigating students' fractional understandings with regard to their mathematics achievements; we can mention studies by Aksu (1997), Keijzer (2003), Empson (2003), and Wong and Evans (2007).

In Aksu (1997)'s study, differences in student performance when fractions were presented in the contexts of (a) understanding the meaning of fractions, (b) computations with fractions and (c) solving word problems involving fractions were investigated. A test on fractions which was consisted of three parts was administered to 155 sixth-grade students: a concept test, an operations test, and a problem-solving test. As one of her research questions, she investigated relationships between student performance on these tests and previous mathematics achievement. Analysis results showed that there were significant differences in performance on these three tests according to previous mathematics achievement.

In the scope of his study mentioned earlier, Keijzer (2003) also carried out a case study to offer an in-depth analysis of the fraction learning process of a low achieving student in the context of Realistic Mathematics Education. He concluded that although low-achievers benefited from experimental instruction, they also experienced difficulties in the formalization process with regard to fractions.

Empson (2003) analyzed two low-performing students' experiences in a first-grade classroom oriented toward teaching mathematics for

understanding. She investigated the nature of students' participation in classroom discourse about fractions. Based on transcripts of interviews that were administered before and after the instruction, and analyses of classroom interactions, Empson attributed these two students' success to three factors: Use of tasks that elicited the students' prior understanding, creation of a variety of participant frameworks, and frequency of opportunities for identity-enhancing instruction.

Wong and Evans (2007) investigated students' conceptual understanding of equivalent fractions by examining third, fourth and fifth grade students' responses to questions using symbolic and pictorial representations. Students were administered general mathematics achievement and fraction tests. The responses of students with limited general mathematics achievement were compared to those of their more competent peers. As a result, the differences that emerged between the two groups in their conceptual understanding of equivalent fractions were highlighted.

Although there is a vast body of experimental studies on teaching of fractions, what this brief review of research literature points out is that there is a need for a study making an in depth analysis about effects of an intervention on low, middle and high achieving students' fractional understandings. As a result, we combined Streefland (1991)'s study including domain of fractions with Woodward and Baxter (1997)'s study which does not deal with fractions but separates students into three achievement categories. Based on the literature reviewed above, we determined the research questions as follows: Does an experimental learning environment starting with equal distribution and sharing situations, and focusing on group and class discussions have different effects on fractional understandings of low, middle, and high achieving students? What are the results when the effects of this learning environment are compared to that of traditional learning environment that learning is a solitary activity and directed toward rules and algorithms?

METHODOLOGY

Instruments

Three instruments were used in the present study: General Mathematical Achievement Test (GMAT), Fractional Understanding Pre Test (FUPRT), and Fractional Understanding Post Test (FUPT). Consisting of 25 multiple-choice questions, the GMAT was used to measure basic

domain knowledge of students and to determine the achievement level of students. For latter aim, only the number of right answers of each student were taken into account.

The FUPRT included 6 open-ended and 2 multiple choice questions while FUPT consisted of 7 open-ended and 1 multiple-choice questions. Instead of bare computations or conventional word problems, all questions in the FUPRT and FUPT were represented in contexts exemplifying real-life situations. For most of the open-ended questions, students were asked to make drawings about the contexts of these questions. For multiple-choice questions, additional written explanations about students' choices were asked; that is, the thinking processes of children were as important as their answers.

The FUPRT and FUPT were not completely same, but we kept them structurally parallel as seen in the following sample questions from these tests and in Table 1.

Table 1: Common Concepts or Skills That Were Expected to be measured by the Parallel Questions in FUPT and FUPRT

Questions	*Common concepts or skills from the FUPRT and FUPT tests*
1	Part-whole relationship
2	Expressing results of equal sharing situations by using fractions
3	Comparing fractions by referencing a whole number or another fraction
4	Finding unit fraction and comparing given fractions by using unit fraction
5	Writing share of each person with fractions in non-equal sharing situations.
6	Finding fraction of a given fraction
7	Comparing given fractions by using ratio interpretation of fractions
8	Grasping magnitude of fractions and developing strategies to compare

FUPRT 6. "Elif's father brought chocolate for her. On the first day, she ate 1/3 of whole chocolate. On the second day, she ate half of the chocolate that she had eaten on the first day. Show the chocolate that Elif ate on the second day by drawing pictures and express what fraction of the whole chocolate it is."

FUPT 6. "A runner ran 1/4 of a way on the first day. On the second day, he ran 1/3 of the way that he had run on the first day. Show the way that runner ran on the second day by drawing pictures and express what fraction of the whole way it is."

Questions in the FUPRT and FUPT were prepared by the researchers of the current study in the framework that have been included in the primary school mathematics curricula of Turkey for fraction teaching at 4th and 5th grade levels. Moreover, questions used in Hart (1981)'s research, activities used in Streefland (1991)'s and Keijzer (2003)'s experimental study, and explanations about teaching of fractions in the Principles and Standards for School Mathematics (NCTM, 2000) provided an important basis for question preparing process.

Selection of Participants

Two different sex-mixed elementary schools in Bursa/Turkey were chosen as experimental and control schools. Determinative factors in selecting these schools were positive attitudes and open mindedness of management board and teachers toward this kind of research. There were two 4th and two 5th classes in the experimental school and one of each of them were randomly selected as experimental group. In the control school, there were four 4th and four 5th classes. The GMAT, FUPRT and FUPT were administered to all students in these classes. And from this "data pool", students in the control group were selected by considering equality of their GMAT scores with that of students in the experimental group. The control and experimental groups were consisted of 55 students for each and numbers of the low, middle and high achieving students were close to each other (Table 2).

Table 2: The GMAT Score Ranges and Numbers of Students for Each Achievement Level

Group	Grade	Achievement level	Range of GMAT score	Numbers of students
Experimental	4	High	23-16	10
		Middle	15-11	9
		Low	10-6	8
	5	High	20-14	10
		Middle	13-10	9
		Low	9-6	9
Control	4	High	25-15	8
		Middle	14-11	11
		Low	10-6	8
	5	High	23-15	9
		Middle	14-10	9
		Low	9-5	10

As seen in Table 2, range of the GMAT scores for each achievement level is not completely similar at each grade in the experimental and control groups. Because, we established the highest and the lowest scores to hold a view about general achievement levels of each grades at first. Then, we defined the levels based on these scores.

Experimental Instruction

During instruction carried out by the first author, 15 activities were designed according to the basic principles of theories of Socio-constructivism and Realistic Mathematics Education; these activities were separately implemented with each grade in the scope of their math lessons. Same activities were used for the fourth and fifth graders, and their implementation lasted 16 lessons, approximately 7 weeks. We took the time into consideration allocated by primary school math curricula for teaching of fractions to determine the duration of experimental instruction. Therefore, experimental group did not have any advantage in point of number of lessons involving fractions when compared to control group. We did not intervene in the instruction given to the control group, but we made observations during their math classes and interviewed with their classroom teachers occasionally. From that perspective, we made sure they were attending classes on their regular lessons in a traditional manner.

As seen from the sample in Appendix, activities used in the present study mostly focused on posing story problems that were real or could be real, and children were encouraged to solve these problems using their own strategies. Discussion of the problems directed to understanding children's thinking, comparing strategies and resolving disagreements or ambiguous mathematical claims. In addition to these, working in groups that consisted of 3 or 4 students, group and class discussions, sharing ideas with the peers in the group and whole class, and the researcher were other prominent properties of learning environment. We preferred this kind of intervention because many studies showed that such learning environments led to greater mathematical understanding and problem solving achievement, especially for middle and high achieving students (Boaler, 1998; Fuson et al., 2000). During the instruction, the researcher took roles such as presenting the problem situations, encouraging children to express their solutions without worrying about making mistakes, guiding and giving hints to students when needed, and leading discussions.

As Charalambous and Pitta-Pantazi (2007) highlighted their study, concept of fractions consists of five interrelated subconstructs: part-whole, ratio, quotient, measure, and operator. Our activities were arranged based on

the first three subscontructs. Measure and operator subscontructs were omitted since they are more related operations with fractions that were not involved in this study. Besides, the first two activities of the instruction were about equal sharing and distribution situations. This category of problems was chosen as the starting point because there were many documents which pointed out that children could invent strategies to solve them without direct instruction, using informal knowledge of division by distribution and repeated halving (Galen et al., 2008; Hunting and Sharpley, 1991; Pithkethly and Hunting, 1996; Charles and Nason, 2000; Smith, 2002; Streefland, 1991). As to other activities, their purpose was to help children to acquire basic fractional concepts and procedures such as unit fractions, equal fractions, comparing fractions.

Data Analysis Procedures

Before the experimental instruction, first version of the GMAT including 30 multiple choice items was administered to all fourth and fifth grade students in another primary school to evaluate its reliability. All answers of this test were scored as 1 or 0 (true or false), and according to reliability analysis results, 5 questions were extracted from the test. Croanbach's alfa coefficient of the last version was 0.71.

All answers of each question of the FUPRT and FUPT were coded as 0 (not correct), 1 (partly correct), 2 (almost correct) and 3 (completely correct). Therefore, the highest total score that a student could get from these tests was 24. Based on this coding system, Croanbach's alfa coefficients of these tests were found as 0.80 and 0.78, respectively. Then, means of the FUPRT and FUPT scores were computed separately for each achievement level in both groups. Finally, independent samples t-test and Analysis of Covariance (ANCOVA) were respectively used to find whether there were significant differences among means and mean changes of the low, middle and high achieving students in the experimental and control group.

FINDINGS

In Figure 1, we presented the changes of means from the FUPRT to FUPT with regard to achievement levels and grades in the experimental and control group.

We can mention five remarkable outcomes that can be derived from graphs in Figure 1. First of them is declining of the means at each grade and achievement level in the control group. The second one is decreasing in the mean of the low achieving students in the fourth grade of the experimental

group after instruction on the contrary to the other achievement levels. The third result is that mean of the low achieving students are higher than that of the middle achieving students in the fifth grade of the experimental group before and after instruction. In addition, the middle achieving students in the fourth grade of the control group have a higher mean than the high achieving students before and after instruction. Finally, lines representing mean changes of the high, middle and low achieving students are almost parallel at most of the grade levels.

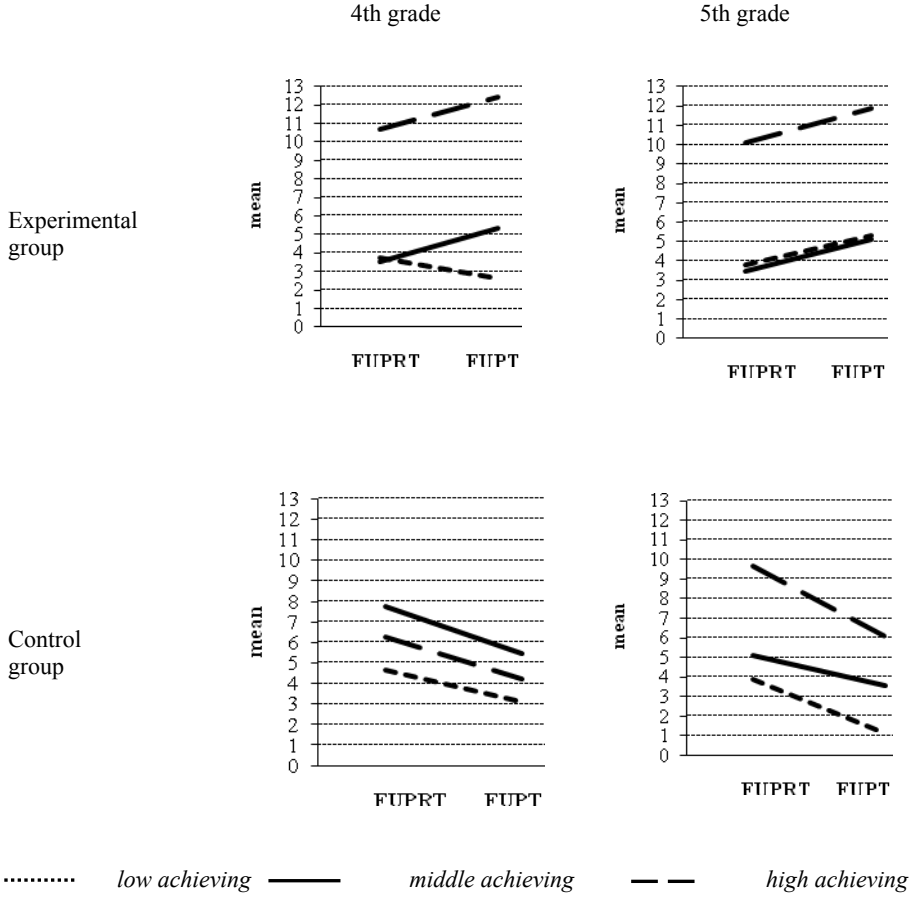


Figure 1: FUPRT and FUPRT Means of Students in the Experimental and Control Group Based on Their Achievement Levels

In Table 3 and 4, descriptive statistics and independent samples t-tests results regarding the FUPRT and FUPT scores are presented for each achievement level.

As shown in Table 3, most of the students' means in the experimental group are lower than that of the control group before instruction. There are only two exceptions: high achieving students in the fourth and fifth grades. Moreover, t values in Table 3 show that there are no significant differences found between the FUPRT means at each achievement and grade level except middle achieving students in both groups.

Table 3: Descriptive Statistics and İndependent Samples t-tests Results Relating FUPRT Scores

		Experimental Group			***ControlGroup***			***t***	***p value***
		n	$\bar{x}$	***SS***	***n***	$\bar{x}$	***SS***		
4th grade	***High achieving***	**10**	**10.7**	**5.01**	8	6.25	6.39	*1.66*	*0.12*
	Middle achieving	*9*	*3.56*	*2.83*	11	7.73	4.29	**-2.50**	**0.02***
	Low achieving	**8**	**3.75**	**2.92**	8	4.63	2.56	**-0.64**	**0.53**
5th grade	***High achieving***	10	10.1	4.25	10	9.70	5.60	*0.18*	*0.86*
	Middle achieving	9	3.44	2.83	9	5.11	2.62	**-1.30**	**0.21**
	Low achieving	9	3.78	3.90	9	3.89	3.41	**-0.06**	**0.95**

* Significant at 95% level

Table 4: Descriptive Statistics and Independent Samples t- tests Results Relating FUPT Scores

		Experimental Group			***Control Group***			***t***	***p value***
		n	$\bar{x}$	***SS***	***n***	$\bar{x}$	***SS***		
4th grade	***High achieving***	10	12.4	5.56	8	4.25	5.31	*3.15*	*0.006**
	Middle achieving	9	5.33	3.91	11	5.46	3.33	**-0.08**	**0.94**
	Low achieving	8	2.63	2.97	8	3.13	2.95	**-0.34**	**0.74**
5th grade	***High achieving***	10	11.9	5.61	10	6.10	6.57	**2.12**	**0.048***
	Middle achieving	9	5.11	3.79	9	3.56	2.60	**1.01**	**0.32**
	Low achieving	9	5.33	4.58	9	1.11	1.36	**2.64**	**0.02***

* Significant at 95% level

As seen in Table 4, on the contrary to the FUPRT, students in the experimental group have higher means than the control group after instruction except the middle and low achievers in the 4th grade. t values indicate that differences between the FUPT means are significant at the high achievement levels in the fourth and fifth grades and at the low achievement level in the fifth grade.

Univariate ANCOVAs were separately conducted for experimental and control group using the low, middle and high achieving students' mean scores in the FUPRT as covariates. So, dependent variables were low, middle and high achieving students' mean scores in the FUPT.

For each class in the experimental group, ANCOVA results indicated that FUPRT mean scores of students at each achievement level were feasible covariates (for fourth grade $F(2, 21) = 0.516$, $p>0.05$; for fifth grade, $F(2, 22) = 0.2$, $p>0.05$). Adjusted means for high, middle and low achievement levels in fourth grade were 9.28, 7.23, and 4.39, respectively. In fifth grade, these values were 8.62, 7.07, and 7.03. The differences among adjusted means were not significant (for fourth grade $F(2, 21) = 2.916$, $p>0.05$; for fifth grade, $F(2, 22) = 0.308$, $p>0.05$), meaning that there were not significant mean changes from FUPRT to FUPT at each achievement level in the experimental group.

As for the control group, ANCOVA results showed again that FUPRT mean scores of students at each achievement level could be used as covariates (for fourth grade $F(2, 21) = 1.474$, $p>0.05$; for fifth grade, $F(2, 22) = 3.387$, $p>0.05$). Adjusted means for high, middle and low achievement levels in fourth grade were 4.33, 4.53, and 4.31, respectively. In fifth grade, these values were 3.79, 4.42, and 2.82. There were insignificant differences in mean changes from FUPRT to FUPT at each achievement level in the control group as well (for fourth grade $F(2, 21) = 0.023$, $p>0.05$; for fifth grade, $F(2, 22) = 0.539$, $p>0.05$).

DISCUSSION

The purpose of this study was to investigate whether low, middle and high achieving students could benefit at the same extent from an experimental instruction prepared to provide a more sound fractional understanding to 4th and 5th graders. In addition, we aimed to compare effects of this instruction to that of traditional learning environment. First of all, when we evaluated influence of the designed learning environment regardless the domain of fractions, we found that classroom interaction and

working in group enhanced pupils' comprehension at almost each achievement levels. This finding matched up with the results of studies by Boaler (1998) and Fuson, Carroll, and Drueck (2000).

As to findings concerning fraction learning, means and t-tests results pointed out that most of the students in the experimental group benefited from the instruction as it was found in studies by Streefland (1991) and Keijzer (2003). According to ANCOVA results, the positive effect of the designed learning environment on high, middle and low achieving students was substantially similar. Likewise, the high, middle and low achievers in the control group also did not show any difference with regard to effect of the traditional learning environment on their fractional understandings.

Two indicators show that the high achievers in the experimental group took advantage of the instruction to great extent: noticeable increase in means of these students after instruction and t values which compare their means to that of their correspondents in the control group. Likewise, there are some evidences of the instruction's positive effect on students at middle achievement level. There was a significant difference between the FUPRT means of the middle achieving students in the fourth grade in favor of the control group. However, students at this achievement level in the experimental group succeed in closing this gap after instruction. Additionally, the middle achievers in the fifth grade of the experimental group increased their means while their correspondents in the control group exhibited a regression in point of mean. These results supported Woodward and Baxter (1997)'s findings revealing strong impact of experimental instruction on the high and middle achievers.

The setback of the low achievers in the fourth grade of the experimental group should be considered cautiously. This finding may denote the fact that both questions and activities used in the present study were more formal and incomprehensible for these students. On the contrary, the low achievers in the fifth grade of the experimental group displayed a considerable improvement. Most probably, these students have a more sophisticated maturity level in terms of fractional understanding because of their grade level. However, means of the low achievers in both grades were not satisfactory when compared to the highest mean that could be reached. This means that increase in instructional time is necessary for them so that they understand rudimentary fractional concepts before proceeding to subsequent levels. All findings about the low achievers in the current study are similar to the findings of studies by Aksu (1997), Keijzer (2003), Empson (2003), and Wong and Evans (2007).

Generally speaking, results of this study were in favor of students in the experimental group. But there would be more favorable results. If the low achieving students in the experimental group had reached a mean that was closer to the mean of students at the other achievement levels after instruction, we could be more confident about success of our instruction. Likewise, the conspicuous differences between the FUPRT means of the high achievers and the other students in the experimental group remained unchanged at the end of the instruction. If this gap could have diminished, we would be more satisfied with our results.

Two interesting results found in this study were about low achievers in the fifth grade of the experimental group and middle achievers in the fourth grade of the control group. They were more successful than the students at the subsequent level. These findings may be attributed to structure of the questions used in the FUPRT and FUPT. Because these questions require to think on underlying concepts rather than to apply algorithms and rules without understanding as traditional system focus on. It appears that some students are more liable to ponder on both what to do and why even if they get lower scores on multiple choice tests like the GMAT used in the present study.

Another striking result was the decrease in means of the students in the control group. This decrease was observed at all grade and achievement levels. This case may point out increase in questions' difficulty levels from FUPRT to FUPT. Although questions in these two tests were structurally parallel, used numbers and fractions in questions of the FUPT may have complicated it.

This research leaves some questions for future research such as follows: If a longitudinal study that has the same conditions with the present study had been carried out, would the results be more beneficial for students at all achievement levels? As we noticed, low achievers seem to benefit less from an educational setting in which fractional concepts are topics in whole-class discussion. How can we improve our learning environment described in this study in order to enhance fractional understandings of students who are less proficient in mathematics? How can be qualitative data used to have more information about effects of our instruction? What if a learning environment similar to that of us is designed for second and third graders? It is obvious that more studies are needed to answer questions posed by the present study.

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Appendix

Activity sample: Equally sharing or distributing situations

Aim: To have students solve problems that require sharing or distributing equally and express the results as fractions.

Material: A picture of "Sizinkiler" (it means "Yours" in English) family

- The Picture of Sizinkiler family will be shown and students will be asked: "Do you know this family?". If there is a student who knows, he/she will be asked to tell the rest of the class. If not, I will introduce the members of the family (Babisko, Cıt Cıt, Zeytin and Limon) and explain that events that this family experiences will be told during the lessons.

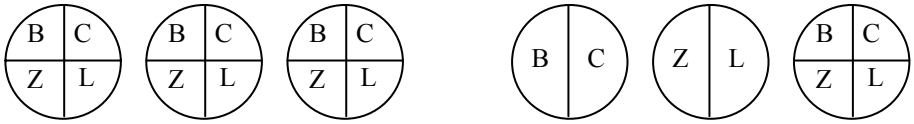
- The first story will be presented: "Sizinkiler goes to a "pide"(a Turkish food like pizza) restaurant for dinner. But 4 "pide"s seems big for them and they order 3 "pide"s instead of 4. In your opinion, how can the waiter serve 3 "pide"s for 4 people? Can you draw the "pide" that each person gets?" Empty papers will be given to the groups and students will be expected to make drawings. While they are drawing, they will be negotiating, asking questions and giving some clues if they need them. (For example: Does each person get a whole "pide"? In how many parts can you divide "pide"s? What can you do to present each person's part?...etc). I will gather the papers and the story will continue as follows:

- "At the table next to the Sizinkiler, there is another group that consists of 9 people. They order 6 "pide"s. Now, again show each person's part with pictures."

- Pupils will be asked to create their own similar stories and to solve them. I will collect the stories and read them out in front of class.

- The students' solutions to the first two problems will be drawn on the board and discussed.

For example:



(The existence of different answers for a problem can be thought as an advantageous situation: the first figure shows $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$, and the other shows $\frac{1}{2}+\frac{1}{4}$.)

- "Can you express the results with fractions?" I will ask this question and we will examine and discuss the results.