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Mathematical Language Used in the Teaching of Three Dimensional Objects: The Prism Example

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This study aimed to determine how students internalize the mathematical concepts taught in their mathematics class, what kinds of differences there are between mathematical languages of the students and the teachers in defining them, and what mistakes the students make in expressing the concepts by using mathematical language. The study was conducted with two mathematics teachers and 35 sixth-grade students in two elementary schools. The data were collected with a classroom observation form and two open-ended knowledge tests. During the data analysis, classroom observation forms were subjected to descriptive analysis and students' knowledge test results were subjected to content analysis. The findings showed that in spite of the differences between what teachers wanted to explain and what students internalized in their minds, these were they were in parallel to one another. However, the students had many problems in expressing their opinions mathematically. The main reasons of these problems derived from students' just focusing a part of definition in coding, having insufficiencies in students' definition skills-inability to use mathematical terminology, using daily life examples like definition and being aware of the exact correspondence of their definitions.

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Introduction

Language is an important tool not only for expressing the meanings that already exist in mind but also for forming new concepts, relations and meanings, and for sharing this formed knowledge (Baki, 2008). The effective and accurate utilization of this tool plays an important role in establishing environment for healthy communication allowing for development of accurate conceptions of idea (Nuhrenborger and Steinbring, 2009). Mathematical language provides a mean for effective discourse that words and symbols unique meanings, and all users of this language infer the same meaning from the same expression (Bali, 2003). As such, mathematics becomes a universal language that enables its users to express scientific thoughts and concepts. The mathematical language, which is formed as a result of the blending of daily language with mathematical concepts, possess a unique technical vocabulary, which renders it different and more complex than the language used in daily life (Austin and Howson, 1979; Raiker, 2002). In an educational setting and in mathematics classrooms, if it is used effectively, what the teacher teaches and what the students learn align well (Yeşildere, 2007). Otherwise, the use of less accurate language potentially leads to misconceptions (Raiker, 2002). The peculiar relationship between mathematics and language has been defined in a way to minimize the potential misconceptions and difficulties to emerge in the process of making sense of mathematics (Adanur, Yağız and Işık, 2004).

With the increased importance attached to the mathematical language in communicating disciplinary knowledge (Boulet, 2007; Cirillo, Bruna and Eisenmann, 2010; Doğan and Güner, 2012; Ferrari, 2004; Rudd, Lambert, Satterwhite and Zaier 2008), the number of studies which address the effects of the knowledge of mathematical concepts to the formation of students' mathematical languages (Capraro and Joffrion, 2006; Çakmak and Bekdemir, 2012; Dur, 2010; Gökbulut, 2010; Korhonen, Linnanmäki and Aunio, 2011; Monroe and Orme, 2002; Morgan, 2005; Raiker, 2002; Vogel, and Huth, 2010; Woods, 2009; Yeşildere, 2007) is on the rise in the literature. Also, it is important that teachers playing an important role in the formation of students' mathematical languages (Mercer and Sams, 2006) should effectively use the mathematical language along with having a command of pedagogy and mathematics. Therefore, investigating the mathematical language used by teachers and students in the conceptual development of mathematical concepts, their comparison and uncovering the differences between two languages are important for the development of this language.

A relevant study in the literature (e.g., Moschkovich , 2007) compared the definitions of students and teachers with regards to parallelogram and trapezoid. The study showed while the teachers mostly used formal definitions by using mathematical language, the students used informal definitions by using daily life expressions in their definitions. In another study (e.g., Huang, Normandia and Greer, 2005), how teachers and students developed a conceptual understanding of knowledge and differences between these processes were researched in the study about elementary school students and teachers. The study showed that knowledge was not directly transmitted to the student from teacher and there were differences between teachers' and students' mathematical language. Also another study (Raiker, 2002) researched whether teachers and students gave different meaning to mathematical concepts or not. The findings of the study showed that teachers and students defined the concepts differently because of their actual positions. Considering these studies, determining the barriers in front of the mathematical language which is one of the important elements of mathematical communication is also important for teaching mathematics.

The use of mathematical language pertaining to the topic of prism was examined in this study for the following two main reasons: i) Along with basic mathematical concepts (addition, subtraction etc.), the necessity of presenting many other concepts pertaining to the subject (lateral face, space diagonal etc.) by the teacher with an appropriate mathematical language while teaching the subject of prism, ii) Ability to measure the levels of teachers and students to establish effective communication by using examples, which are easily-found in students' everyday environments (Gökbulut, 2010), on the subject of prism.

The fact that the mathematical language and mathematical concepts are of abstract structures is of importance for analyzing the mathematical languages of secondary school students, who are at the stage of dealing with abstract operations (Senemoğlu, 2011, p. 49). At the time of the study, the sixth-grade students had already gone through a five-year communication process, which the classroom teachers had established and with which the students had been familiar. At this step, mathematics teachers are expected to explain more concepts and to use the mathematical language more effectively. Therefore, it was thought that whether a mathematics teacher and a student use the mathematical language effectively or not could best be examined at the level of sixth-grade. On the other hand, the reason the second semester of the academic year was selected is to minimize as much as possible the mistakes arising from the process of getting used to the secondary school and to the teacher's language.

Considering these issues, the purpose of the study was to determine how sixth-grade students internalize in the concepts explained using the mathematical language on the subject of learning prisms, which is one of the sub-domains of "Geometrical Objects" of the learning domain in Sixth-Grade, how teachers and students explain these concepts by using the mathematical language and which mistakes students do in this process.

The main focus in the study is to determine the differences between mathematics teachers' mathematical language use in teaching the concepts as part of the secondary school sixth-grade subject of "Let's learn prisms" and students' mathematical language use and to define students' mistakes in this process.

Three specific questions guided our data collection and analysis process.

1. Are what teacher say (or what he/she tries to establish) and what students construct in their minds the same?
2. Which differences are there between the mathematical language used by teachers and students?
3. What kind of mistakes do students make in mathematical language use while expressing mathematical concepts?

Method

1. Model

A case study method from qualitative research methods which allows for the thorough analysis of one or more events, media, programs, social groups or other interconnected systems (McMillan, 2000) was used in this study.

2. Participants

The research was carried out at the second semester of the 2011-2012 Academic Year with two mathematics teachers and 35 sixth-grade students (11 years) in two public secondary schools located in a medium-sized Eastern Mediterranean city in Turkey. 19 of these students were educated in the state schools in the city centre and 16 of them were educated in the village schools. The first teacher working in the city centre was coded as T1and had a 10- year experience in teaching. The second teacher working in the village was coded as T2 and had a 5-year experience in teaching. A purposeful sampling method of typical case sampling was used for selection of participants. The participant teachers were coded as T1 and T2. The participant students correspondent with teachers' codes (e.g. students of T1 were coded as T1S1,…,T1S19; and students of T2 were coded as T2S1,…,T2S16).

3. Data Collection Instruments and Collection of Data

Data were collected in two stages. First, the mathematical concepts and their definitions on the subject of prism which were taught during two course hours were collected using the observation form. The aim here was to determine the concepts that the teacher used in the communication process and how these concepts were expressed. At the second stage, data on students' thinking were collected using two open-ended knowledge tests –KT1 and KT2-, which were on the concepts taught as part of the subject of prism for sixth-grade students. These tests were prepared after taking opinions from three educators who had command of the subject of qualitative research. The question asked in KT1 is the following: Suppose that your best friend missed the class the day the subject of prism was taught. This is why he/she does not know anything about what the teacher lectured in the class. Convey the lecture to your friend. On the other hand, the question asked in KT2 is the following: Define the concepts of lateral face, edge, space diagonal, prism, vertice, base and height; and show the elements of a prism by drawing one in the boxes. Students were firstly given KT1, and after collecting KT1 test forms back, KT2 test forms were handed out, in order to prevent KT2 from affecting students' responses to KT1, as KT2 involved elements of concepts given in KT1.

4. Analysis of Data

Analysis of data was performed in two stages: analysis of the observation form and KT1, and analysis of KT2. At the first stage, analysis of the data pertaining to the first sub-problem was performed using the observation form and KT1. Data extracted from the observation form were subjected to descriptive analysis in which the conceptual structure is known beforehand (Yıldırım and Şimşek, 2008) and then, through the observation data obtained, the concepts mentioned by the teacher in the class were determined. Besides, the data were analyzed by considering teachers' emphasis on certain characteristics of concepts and their dialogues with students. Two independent raters coded the observation data. Points that were thought to have influence on concepts were determined through the experts' consensus, and the analyses of teachers' lectures were summarized. In order to identify the concepts and relations that could explain the data collected through KT1, the data were subjected to content analysis. Students' responses to KT1 were analyzed by two experts, and the similarities and differences between the analyses of teachers' classes and the responses given by students to KT1 were noted.

At the second stage, KT2 results were analysed so to respond the second and third sub-problems. The coding categories reached

1) Accurate drawing of prism and correct use of mathematical language;
2) Accurate drawing of prism but inaccurate use of mathematical language;
3) Accurate drawing of prism but incorrect use of mathematical language;
4) Accurate drawing of prism but no definition;
5) Completely incorrect answers or blank ones. Frequencies and percentages of student responses were determined. At the same time, at this stage, students' mistakes were categorized. To increase the reliability of coding; parts that had been removed, categories that had been defined and expressions that had been put under them were re-analyzed by a group, which was consisted of the two experts and another expert from outside the scope of the study, and then common responses were identified. These three experts discussed the responses and added new categories if needed. Lastly supportive qualitative data including quotes were used to explain the sources of errors.

Findings

1. Findings related to the sub-problem "Are what teacher say (or what he/she tries to establish) and what students construct in their minds the same": In order to determine what the teacher says (or what he/she tries to establish); the two participants teachers' lectures are summarized (below) including the durations they dedicated to each concept. The reason a special emphasis was put on durations is the idea that time is an important factor influencing students' comprehension of concepts.

Class analysis of T1:

The teacher started the class by sharing an example from daily life: "Folks! The other day I received a box of books I had ordered earlier. They put the books in the box like this and sealed the box like this... So, what is this box?" By asking this question, he let the students figure out that the box is a prism. Then, he talked about examples of prisms encountered in daily life. This opening discussion lasted about 11 minutes. He then introduced the elements of a prism on a teaching object and provided definitions. This lasted approximately 6 minutes. He then offered examples of objects that are not prisms, formally defined prisms, and finally asked students to draw a prism and to share its definition (approximately 10 minutes). While discussing examples of prisms and their elements, he concluded the lesson by asking and asked students to draw expanded rectangles and triangular prisms (6 minutes). He started the second class by discussing four space diagonals about 10 minutes, and then he provided a formal definition by asking students to visualize the two remotest corners of the room. He continued the class in the Q&A style by describing oblique and right prisms (5 minutes). While describing oblique and right prisms, he both used teaching object and drew examples on the board. In the remaining time, he presented examples from the computer and from the course book.

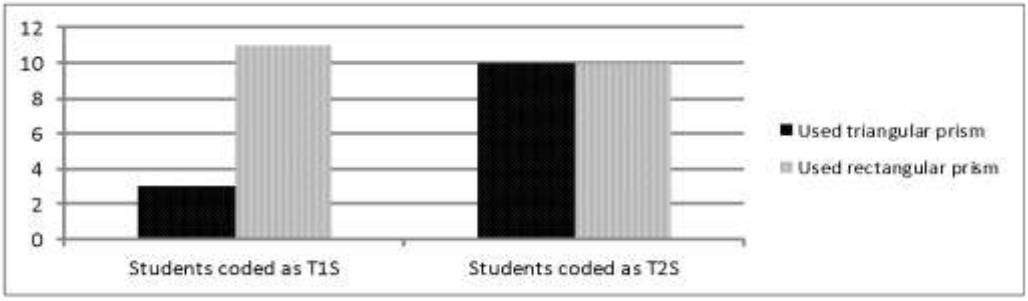
Class analysis of T2:

The teacher spent the majority of the first session by trying to learn what students know in order to determine students' prior knowledge on the subject. In this process, he used triangular prism and rectangular prism together, and introduced many elements of prisms. He put a special emphasis, for around 7 minutes, on the concept of dimension. He stated, "the most important concept in prisms is the concept of height, which is the third dimension". T2 used a matchbox as an example, which was a small object considering the size of the classroom. Using the matchbox; he introduced the concepts of height, vertix and edge along

with the number of faces for around 3 minutes. Then, he talked for 4 minutes about the concepts of oblique prism and right prism. While describing oblique and right prisms, he did not use teaching object and only drew examples on the board. He devoted 8 minutes to the concept of height, which he thinks the most important concept in prisms. The first session ended following a dialogue between a student and the teacher. He started the second session by discussing the subject of prisms (2 minutes). Using the Q&A method, he asked T2 students to calculate the perimeter of the lateral faces of the rectangular prism that he had shown in the previous session, and then he talked about the perimeters these faces (around 6 minutes). While defining lateral face, he said that "the lateral face of a prism is always a rectangle" and demonstrated it on the material (for around 5 minutes). He dedicated 2 minutes to the concept of edge by also showing it on the example he had. After talking for 7 minutes about the concepts of diagonal and space diagonal, he summarized the subject of prisms for around 3 minutes and then asked students to write down these concepts on their notebooks.

In order to determine what students construct in their minds, as stated in the first sub-problem, the responses given by the students of both teachers to the KT1 were analyzed. It was observed that while some students used one type of prism, others used two and some others used none. Graph I illustrates the types of prisms produced by the students.

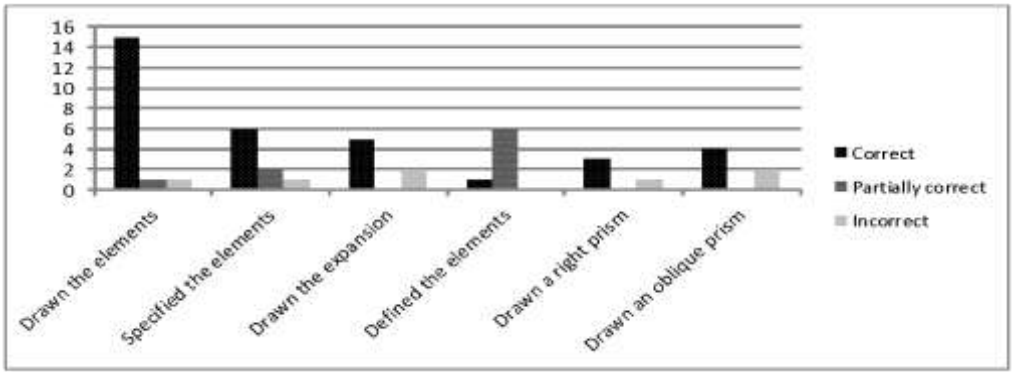
Graph I. Types of prisms used by students coded as T1S and T2S while depicting prisms.



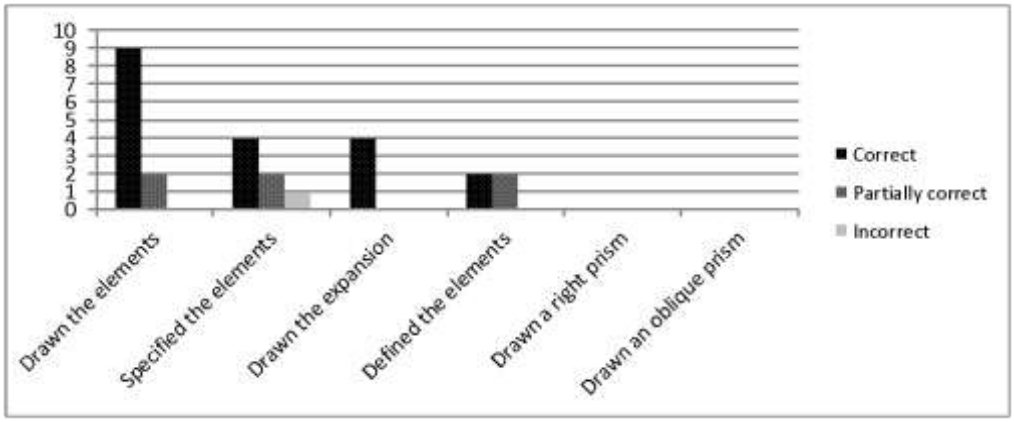
As presented in Graph I, while only three students identified triangular prism, which is less frequently experienced in daily life and textbooks in T1's class, this number was higher in T2's class. This result might stem from the fact that T1 emphasized rectangular prism in his lecture and referenced other types of prisms less frequently. On the other hand, T2 used examples of rectangular and triangular prisms together. Additionally, although both teachers had introduced other types of prisms such as hexagonal and pentagonal prisms in their lessons, none of the students provided such examples.

In Graph II and Graph III. Illustrate the approaches that the students used in response to the question in KT1 (Suppose that your best friend missed the class the day the subject of prism was taught. This is why he/she does not know anything about what the teacher lectured in the class. Convey the lecture to your friend.)

Graph II. Depiction codes and frequencies of students coded T1S for the subject of prisms



Graph III. Depiction codes and frequencies of students coded T2S for the subject of prisms



The depiction codes and frequencies of T1S are given in Graph II. They followed the following way while depicting prisms: Demonstrating by drawing the elements (15 correct, 1 partially correct, 1 incorrect), by specifying the elements (6 correct, 2 partially correct, 1 incorrect), by drawing the expansion (5 correct, 2 incorrect), by defining the elements (1 correct, 6 partially correct), by drawing a right prism (3 correct, 1 incorrect) and by drawing an oblique prism (4 correct, 2 incorrect).

The depiction codes and frequencies of T2S are given in Graph II. They followed the following path while depicting the subject of prisms: Demonstrating by drawing the elements (9 correct, 2 partially correct), by specifying the elements (4 correct, 2 partially correct, 1 incorrect), by drawing the expansion (4 correct) and by defining the elements (2 correct, 2 partially correct). It is notable that they did not attempt to draw a right prism or an oblique prism.

As shown that students mostly preferred to depict prisms by either drawing or specifying the elements of a prism. It was also observed that the instances of defining these elements by using a mathematical language were lower in the class of T2. This might be due to the fact that T1 dedicated more time and energy to defining the elements of prisms in his classes and asked students to write down those definitions. On the other hand, T2 asked different questions and dedicated less time.

Note also that none of the students in T2 class used oblique or right prisms in their depictions. On the other hand, T1's students used examples of these types of prisms. This

finding required a revisiting of class observations, which indicated that both teachers dedicated approximately same amount of time to introducing these types of prisms. However, T1 both used teaching objects and drew examples on the board. On the other hand, T2 only drew examples on the board. Differences in students' responses might be attributed to these teacher practices.

2. Findings related to the sub-problem "Which differences are there between the mathematical language used by teachers and students?": With the purpose of finding answers to the second sub-problem; class observations and students' responses to KT2 were analyzed. With the purpose of determining whether students managed to define the concept of prism, the visual illustrations and verbal definitions provided by students were coded in line with the data obtained from KT2 (Define the concepts of prism, and show by drawing a prism in the boxes) and these codes and frequencies are presented together in Table 1.

Table 1. Categories and frequencies concerning students' definitions of the concept of prism

Prism	***T1S***		***T2S***	
	f	***Example***	***f***	***Example***
Accurate drawing of prism and correct use of mathematical language	**0**	-	**1**	*"Prisms are three dimensional figures with a height and equal upper and lower bases"*
Accurate drawing of prism but inaccurate use of mathematical language	**10**	*"An empty box"*	**2**	*"Three dimensional objects"*
Accurate drawing of prism but incorrect use of mathematical language	**4**	*"Prism forms of a square, triangle and rectangle"*	**1**	*"A prism has three faces"*
Accurate drawing of prism but no definition	**3**	-	**11**	-
Totally incorrect or empty answer	**2**	-	**1**	-
Total	**19**		**16**	

17 of T1S students failed to define prism using a correct mathematical language although they presented accurate drawings. While ten of these students provided a definition using an inaccurate mathematical language, three of them did not give any definition at all. 15 of T2S students drew prisms accurately, only one of them provided a nearly correct definition, two used the mathematical language inaccurately and eleven did not give any definition at all.

Similar results were observed among students' responses to KT2 (Define the concepts of lateral face, edge, space diagonal, vertice, base and height; and show the elements of a prism by drawing one in the boxes), and they are shown in Table 2.

Table 2. Categories and frequencies concerning students' definitions of elements of prism

	Frequencies of T1S (f)	*Frequencies of T2S (f)*

Themes / ***Elements of Prism***	*Base*	*Edge*	*Lateral Face*	*Height*	*Vertice*	*Space diagonal*	***Total and Percentage***	*Base*	*Edge*	*Lateral Face*	*Height*	*Vertice*	*Space diagonal*	***Total and Percentage***
Accurate drawing and correct use of mathematical language	9	0	1	0	6	0	**16 (14%)**	3	2	4	2	2	1	**14 (15%)**
Accurate drawing but inaccurate use of mathematical language	6	12	7	13	5	10	**53 (47%)**	2	3	1	2	1	2	**11 (12%)**
Accurate drawing but incorrect use of mathematical language	2	5	4	2	3	0	**16 (14%)**	0	0	0	1	0	3	**4 (4%)**
Accurate drawing but no definition	0	0	1	0	4	1	**6 (5%)**	9	7	4	8	9	3	**40 (41%)**
Totally incorrect or empty answer	2	2	6	4	1	8	**23 (20%)**	2	4	7	3	4	7	**27 (28%)**

As presented in Table 2, the mathematical language was used correctly in 14% of the responses regarding the elements of prisms in T1's classroom, whereas it was used inaccurately in 47% of them and incorrectly in 14% of those responses. On the other hand, 20% of the responses did not include any definition at all. In the classroom of T2; while defining the elements of prisms, 15% of students used the mathematical language correctly, 12% used it inaccurately, 4% used it incorrectly, 41% provided accurate drawings but no definition, and 28% provided no definition at all.

The low percentages in terms of correctly using the mathematical language might have stemmed from students' previous learning. However, obtaining such low percentages after the participants teachers had taught the subject is an unexpected result for this study. Besides, it is notable that there was no student who provided a correct definition but no drawing. This shows that students who are capable of correctly defining the concept are also capable of drawing it. On the other hand, it could be stated based on the findings that every student who can draw is not necessarily capable of providing a definition.

3. Findings related to the sub-problem "What kind of mistakes do students make in mathematical language use while expressing mathematical concepts?: As was also presented in the first sub-problem; parallelisms between the ways teachers taught the course and the responses given by students to KT1 are indicators of the impacts of the communication during classes upon students. However, it cannot be stated that there are parallelisms between the responses that students gave to KT2 and the definitions provided by teachers in class. The incorrect responses given by students to KT2 were group under the following headings:

- Students coded only a part of the definition
- Insufficiencies in students' definition skills
- Inability to use a mathematical terminology

- Use of daily life examples as definitions
- Students' unawareness about the exact correspondence of their definitions
- Use of a particular case introduced in the class as the definition

➢ *Students coded only a part of the definition:*

While listening to teachers; some students perceived, selected or coded in mind only a portion of the definition provided by the teacher. For example; T1S12, T1S13 and T2S10 gave the following definitions.

T1S12: "An object whose all sides are closed." (Prism)

T1S13: "Surface of a prism" (Lateral face)

T2S0: "Three-dimensional object" (Prism)

T1S12 heard the following definition in the class: "A prism is a closed object that is obtained by bringing together the endpoints of equilaterals of two parallel polygons. However, s/he coded only the part about "closed object".

This may complicate for him/her to differentiate between a prism and other closed objects.

➢ *Insufficiencies in students' definition skills:*

The finding that some students did not provide any definitions although they were asked to indicates that they lack this skill. The following are relevant examples:

T1S5: "Space diagonal: Here is the prism's line diagonal."

T1S5: "Base: Base is the upper and lower sides of prisms. Base is here."

T2S3: "Base: There are 2 bases."

T2S5: "Vertice: It consists of six vertices."

These examples are not proper definitions. It can be stated that this situation is a consequence of students' lack of command of the mathematical language.

➢ *Inability to use a mathematical terminology:*

In order for students to use the mathematical language, they firstly need to have a command of the mathematical terminology and then they should internalize meaningful sentences using these terms. The following are examples in this category:

T1S14: "Space diagonal: The line between its remotest vertices."

T1S7: "Edge: The line that constitutes a prism."

T1S8: "Vertice: The thing that connects lines."

T1S9: "Edge: A plain side of a thing is called edge."

T1S18: "Lateral face: Side of a figure."

Although students had learned the concept of "line segment"; they used concepts like "line", "thing" and "side". Use of such words negatively influences the development of their mathematical language skills.

➢ *Use of daily life examples as definitions:*

When teachers proceed in the class to the mathematical definition after giving examples from daily life, some students perceive that concept only through the daily life example. For example, T1S12 and T1S13 gave the following responses in KT2:

T1S12: "Vertice: Pointy place of a prism"

T1S13: "Prism: An empty box"

Students of T1 defined prism as "an empty box", probably because T1 used the daily life example of box in the beginning of the class. Moreover, T1 taught the concept of vertice by telling a story of someone who hurt himself after running against the corner of a desk. After asking the reason, students gave the response "because it is incisive". This is why, students explained vertice using a daily language.

➢ *Students' unawareness about the exact correspondence of their definitions:*

Definitions made by most students actually involved different meanings, because they used inaccurate mathematical expressions or they failed to fully express their opinions. Moreover, students provide their definitions without being aware of this situation. The following are some examples.

T1S10: "Edge: The line that connects vertices is called edge."

T1S16: "Prism: A figure which is given a name according to its bases and which has volume."

T1S18: "Space diagonal: It divides the object into two."

T2S11: "Height: Length of the steep line"

T2S10: "Height: Edge that connects lower and upper bases"

For example, while stating that "the line that connects vertices is called edge", students are not aware of the fact that a face diagonal or a space diagonal could also be obtained by connecting the vertices. Or, the definition "space diagonal divides the object into two" indicates that the student is not aware of the fact that the object can also be divided into two vertically or horizontally. It is necessary to raise awareness of students about these incorrect definitions.

➢ *Use of a case introduced in the class as the definition:*

Teachers give examples in classes in order to enable students to understand the concept better by generalizing a particular case and using it in the concept's definition. The following are relevant examples:

T2S6: "Space diagonal emerges when it comes from the upper left corner to the lower right corner."

T2S5: "Lateral face generally has a rectangular shape"

Probably because it is easier for students to recall the example given by the teacher, students

use such examples rather than developing their own definitions. This is likely to pave the way for an incorrect internalization of the concept in mind.

Conclusion, Discussion and Suggestions

In this study, we traced the connection between mathematical language used by teachers and their students when describing and defining prisms.

It was observed that the definitions that students made, the examples that students selected and the paths that students followed somewhat paralleled to their teachers' own classroom practices. Students tended to use the examples of prisms that their teachers had illustrated while introducing prisms in their lesson. Moreover, in light of course observation analyses, it was observed that the participant teacher coded T1 spent more time in defining concepts than T2, and thus, students of T1 provided more conceptual definitions in KT1 and KT2 than students of T2. Therefore, it could be stated that the mathematical communication environment, which consists of numerous elements such as materials used by teachers during classes, order they follow while teaching concepts or examples they prefer, has a significant impact upon learners.

Along with these similarities, some difference also emerged. For example, students did not use the different types of prisms that teachers had shown during classes. Similar results were reported in Tsamir, Tirosh and Levenson (2008). The authors found that students use certain frequently-used geometrical objects while expressing their mathematical thoughts. Gökbulut (2010) also reported that pre-service teachers failed to diversify prism examples while teaching the topic. Although T2 explained oblique prism and right prism, none of his students used these concepts.

Differences were observed between the mathematical languages used by the teachers and the students. Although the teachers used correct mathematical language in their lecture, only 14% of T1S students and 15% of T2S students adopted such approach when answering questions. This shows that students of both groups struggled to express their idea using formal mathematical language. This struggle was also reported by Woods (2009), Capraro and Joffrion (2006) and Dur (2010) among elementary school students and by Korhonen, Linnanmäki and Aunio (2011) among high school students. Çakmak and Bekdemir (2012) also reported for university students.

Those students who geometrically provided a correct definition also managed to illustrate the relevant concept. Woods (2009) noted a similar finding by outlining that some students experience problems in expressing their mathematical knowledge despite their success in the field of mathematics.

When the responses under the category "Students coded only a part of the definition" are examined, it was observed that students only noticed and focused on a part of the definition. For example, the concept of prism was named by some students as "a closed object" and "a three-dimensional object". A similar situation has also been reported by Gökbulut (2010). It is thought that this situation might have stemmed from selective perception, which is defined as a communication obstacle in which only certain parts of a message are received (Yazıcı and Gündüz, 2010).

Some students preferred defining concepts by drawing arrows on their drawings rather than verbal expressions. Since these illustrations were mathematically correct, it is interpreted that

they did so not because they lacked knowledge on concepts but because they were unable to express their opinions verbally, and thus these responses were addressed under the category of "Insufficiencies in students' definition skills". Similarly, Gökbulut (2010) reports that pre-service teachers do not possess necessary skills to provide definitions.

Students use terms such as "a thing" instead of a mathematical language, and this was interpreted as students' inability to use a mathematical terminology. A similar finding was obtained by Yeşildere (2007), who conducted a study with pre-service teachers on geometrical concepts.

It was observed that daily life examples were used as definitions (e.g. "prism: empty box"). This finding is in parallel with the finding obtained in study carried out by Baş, Çakmak, Bekdemir and Işık (2012).

In responses addressed under the category "Students' unawareness about the exact correspondence of their definitions"; students defined concepts through generalizations (e.g. "edge: the line that connects vertices") that could also be valid for other concepts. This finding is in parallel with the finding of Gökbulut (2010) that students do not use the critical features of the concept while defining the concept of prism.

Some students used a particular case introduced by their teachers in the class while defining subjects (e.g. "space diagonal: it emerges when it comes from the upper left to the lower right corner"), and such responses were addressed under the category of "Use of a particular case introduced in the class as the definition".

Departing from these findings, it is suggested that teachers' awareness about the problems identified in this study should be raised, and teachers should pay attention to their students' correct use of the mathematical language. On the other hand, future studies should address this issue from different perspectives in order to contribute to the elimination of these problems for students.

In this context, it can be said that teachers' language is influential on the students' language use but this effect is not in a direct way. Therefore, teachers should consider aforementioned difficulties of students while they are using mathematical language in the classrooms.

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